Integrated Physics for SRS
Addendum

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Last revised: 2023-06-11

Contents

1 Introduction

This is an addendum to the main text. The purpose of this text is to expand on the ideas in the main text and to introduce prerequisite material.

2 Properties of numbers

The first thing that we are going to need is some mathematical machinery. We will not be using the precise definitions in this course. They are covered to ensure that everyone has seen the concepts and can use them. Therefore, don't be stressed by the terminology since we won't use it much. **In fact, no content in this section will be directly tested.**

2.1 Identity

2.1.1 Additive identity

The additive identity is the number (b) that when added to number (a) results in the number (a). That is: $a + b = a$. After a little reflection it should be apparent that b must be zero. In fact, zero is the additive identity for all numbers.

2.1.2 Multiplicative identity

The multiplicative identity is the number (b) that when multiplied by number (a) results in number (a). That is: $a \times b = a$. It is hopefully clear that b must be one. In fact, one is the multiplicative identity for all numbers.

2.2 Inverses

The table below contains examples for the definitions in this subsection.

Number	Additive inverse	Multiplicative inverse
10	-10	1/10
5	-5	1/5
3/2	-3/2	2/3
-4	4	-1/4
-1/3	1/3	-3
0	0	Undefined

2.2.1 Additive inverse

The additive inverse of a number (a) is the number (b) such that their sum is zero. Or, written symbolically; $a + b = 0$. After some reflection it may be clear that b must be -a. This is true for all numbers. See the table above for examples.

Also, subtraction can be thought of as adding the additive inverse of a number. See the table below for some examples.

2.2.2 Multiplicative inverse

The multiplicative inverse of a number (a) is the number (b) such that their product is 1. Or, written symbolically, $a \times b = 1$. This one is a little harder to visualize, but it must be that $b = 1/a$. This is true for all numbers except for zero. This is because it is impossible to divide by zero. See the table above for examples.

Also, division can be thought of as multiplying by the multiplicative inverse of the divisor. See the table below for examples.

Subtraction	Addition
5 - 3	5 + (-3)
10 - 2	10 + (-2)
11 - (-4)	11 + 4
10 - $\frac{1}{3}$	10 + (-$\frac{1}{3}$)

Division	Multiplication
$5 \div 3$	$5 \times \frac{1}{3}$
$10 \div 2$	$10 \times \frac{1}{2}$
$11 \div (-4)$	$11 \times \frac{1}{-4}$
$10 \div \frac{1}{3}$	10×3

2.3 Commutativity

Commutativity refers to the idea that given two numbers, it doesn't matter which order you add or multiply them. The answer is the same regardless. We will now consider addition and multiplication separately.

2.3.1 Additive commutativity

According to Fisher, "The sum of two or more numbers is the same in whatever order they may be added, Or, stated symbolically, $a + b = b + a$."[1, pg. 22]. Consider the following examples:

$$3 + 5 = 8 \qquad 5 + 3 = 8$$
$$1 + 4 = 5 \qquad 4 + 1 = 5$$
$$20 + 15 = 35 \qquad 15 + 20 = 35$$

Note that subtraction also follows the same rule, however, we must consider the subtraction as adding a negative number (or the additive inverse of the number subtracted). Hopefully the following examples clarify the matter. Note that the same answer is found in each row.

$$5 - 1 = 5 + (-1) = 4 \qquad (-1) + 5 = 4$$
$$7 - 3 = 7 + (-3) = 4 \qquad (-3) + 7 = 4$$
$$9 - 8 = 9 + (-8) = 1 \qquad (-8) + 9 = 1$$

2.3.2 Multiplicative commutativity

According to Fisher, "The product of two numbers is the same, if either be taken as the multiplier and the other as the multiplicand; or, stated symbolically, $a \times b = b \times a$"[1, pg. 34]. For example, it doesn't matter if we have four groups of eight or eight groups of four. In both cases, we have 32 things. Consider the following examples, note that the same answer is found in each row.

$$4 \times 8 = 32 \qquad 8 \times 4 = 32$$
$$12 \times 10 = 120 \qquad 10 \times 12 = 120$$
$$2 \times 8 = 16 \qquad 8 \times 2 = 16$$

Note that division also follows the same rule, however, we must consider the division as multiplying by the reciprocal of the divisor (or the multiplicative inverse of the divisor). Hopefully the following examples clarify the matter. Note that the same answer is found in each row.

$$2 \div 2 = 2 \times \frac{1}{2} = 1 \qquad \frac{1}{2} \times 2 = 1$$
$$8 \div 4 = 8 \times \frac{1}{4} = 2 \qquad \frac{1}{4} \times 8 = 2$$
$$27 \div 3 = 27 \times \frac{1}{3} = 9 \qquad \frac{1}{3} \times 27 = 9$$
$$9 \div \frac{3}{2} = 9 \times \frac{2}{3} = 6 \qquad \frac{2}{3} \times 9 = 6$$

2.4 Associativity

Associativity states that if we add or multiply three numbers together, it doesn't matter which two we add or multiply first.

2.4.1 Additive Associativity

According to Fisher, "The sum of three or more numbers is the same in whatever way successive numbers are grouped or associated in the process of adding. Or, stated symbolically, $(a + b) + c = a + (b + c)$."[1, pg. 22] Consider the following examples, note that the same answer is found in each row.

$$1 + (2 + 3) = 1 + 5 = 6 \qquad (1 + 2) + 3 = 3 + 3 = 6$$
$$7 + (8 + 2) = 7 + 10 = 17 \qquad (7 + 8) + 2 = 15 + 2 = 17$$
$$2 + (4 + 3) = 2 + 7 = 9 \qquad (2 + 4) + 3 = 6 + 3 = 9$$

Note that subtraction also follows the same rule, however, we must consider the subtraction as adding a negative number (or the additive inverse of the number subtracted). Hopefully the following examples clarify the matter. Note that the same answer is found in each row.

$$5 + (8 - 3) = 5 + (8 + (-3)) = 5 + 5 = 10 \qquad (5 + 8) + (-3) = 13 + (-3) = 10$$
$$7 + (5 - 1) = 7 + (5 + (-1)) = 7 + 4 = 11 \qquad (7 + 5) + (-1) = 12 + (-1) = 11$$

You may have noticed that we only considered subtraction in the parentheses. Using the associative property in a problem like $5 - (2 + 2)$ will require the distributive property. We will cover that in section 2.5.

2.4.2 Multiplicative Associativity

According to Fisher, "The product of three numbers is the same in whichever way two successive numbers are grouped or associated in the process of multiplying; or, stated symbolically, $(a \times b) \times c = a \times (b \times c)$"[1, pg. 36]. Consider the following examples, note that the same answer is found in each row.

$$2 \times (3 \times 4) = 2 \times 12 = 24 \qquad (2 \times 3) \times 4 = 6 \times 4 = 24$$
$$7 \times (4 \times 2) = 7 \times 8 = 56 \qquad (7 \times 4) \times 2 = 28 \times 2 = 56$$
$$9 \times (4 \times 3) = 9 \times 12 = 108 \qquad (9 \times 4) \times 3 = 36 \times 3 = 108$$

2.5 Distributive Property

There are certain mathematical ideas that have to be taken on faith. While undesirable, it is a fact of mathematics. This is an example of such an idea. The distributive property shows two ways to evaluate a number multiplied by a sum. The first way is to complete the sum first and then multiply. The other way involves multiplying the multiplicand by each addend and then adding. Written algebraically: $a \times (b + c) = a \times b + a \times c$. The following examples show how the property can be used. Note that the same answer is found in each row.

$$5 \times (3 + 2) = 5 \times 3 + 5 \times 2 = 15 + 10 = 25 \qquad 5 \times (3 + 2) = 5 \times 5 = 25$$
$$8 \times (4 + 1) = 8 \times 4 + 8 \times 1 = 32 + 8 = 40 \qquad 8 \times (4 + 1) = 8 \times 5 = 40$$
$$2 \times (11 + 2) = 2 \times 11 + 2 \times 2 = 22 + 4 = 26 \qquad 2 \times (11 + 2) = 2 \times 13 = 26$$

This fact is useful in algebra. Because in algebra you may not know one of the addends. This property allows you to solve for the unknown addend. This will be covered later. Another useful area is in mental mathematics. Consider the following problems. While the problem at the start is difficult to do mentally, after the distributive property it is more manageable.

$$3 \times 11 = 3 \times (10 + 1) = 3 \times 10 + 3 \times 1 = 30 + 3 = 33$$
$$5 \times 13 = 5 \times (10 + 3) = 5 \times 10 + 5 \times 3 = 50 + 15 = 65$$
$$8 \times 19 = 8 \times (20 - 1) = 8 \times 20 - 8 \times 1 = 160 - 8 = 152$$

2.6 Scientific Notation

For an introduction to scientific notation, refer to the following site: `https://www.mathsisfun.com/numbers/scientific-notation.html`. For a more advanced explanation, refer to `https://`

phys.libretexts.org/Courses/Tuskegee_University/Algebra_Based_Physics_I/01%3A_Nature_of_Physics/1.02%3A_Scientific_Notation_and_Order_of_Magnitude Note that being able to manipulate scientific notation on a calculator is adequate for the computational portion of this class.
To compare the magnitude of a number in scientific notation, make the following comparisons.

1. Consider the exponents. If one number's exponent is larger than the other number's exponent, then that number is the larger number.

2. If the exponents are the same, then compare the coefficient, the number with the larger coefficient is the larger number.

3 Algebra

In this section we continue developing our mathematical machinery. Like the prior section, **none of the material covered here will be explicitly tested**. However, the content provided here will be necessary to solve upcoming material.

3.1 Variables and Expressions

A variable is typically represented by a letter and it can be thought of as a blank. An expression is set of operations on a variable. For example, consider expression 1. In this example, the statement $x+4$ would be called an expression and the letter x would be a variable. The expression could be read to mean take a number (which we call x) and add four to it. A similar expression can be seen in expression 2. In this expression, we take a number (which we call x) and subtract five from it.

$$x + 4 \tag{1}$$
$$x - 5 \tag{2}$$
$$2 \times x \tag{3}$$
$$2x \tag{4}$$
$$y \div 3 \tag{5}$$
$$y/3 \qquad \frac{y}{3} \tag{6}$$
$$-16t^2 + 2t + 7 \tag{7}$$

Another type of expression is shown in expression 3. In this expression, we take a number (which we have called x) and multiply it by 2. We normally would not write the $\times$ symbol and would instead write it as shown in expression 4. In expression 5, the expression means take a number (which we call y) and divide it by 3. We rarely use the $\div$ symbol in algebra. We would typically use one of the forms shown in expression 6.

Of course, the concept of expressions has been generalized, so complicated expressions have been devised, one such example is shown in expression 7. We will cover how to evaluate this type of statement in section 3.5.

3.2 Evaluating Expressions

Now that we know how to interpret simple expressions, we need to know how to evaluate them. Evaluating an expression means to find what an expressions simplifies to if we know what the variable is. This is done by replacing the variable with the value that we pick or are given. (Recall that a variable can be thought of as a blank.) See the table below for examples.

Expression	x = 0	x = 2	x = -2	x = 5	x = -10
x + 4	0 + 4 = 4	2 + 4 = 6	-2 + 4 = 2	5 + 4 = 9	-10 + 4 = -6
x - 5	0 - 5 = -5	2 - 5 = -3	-2 - 5 = -7	5 - 5 = 0	-10 - 5 = -15
2x	2×0 = 0	2×2 = 4	2×-2 = -4	2×5 = 10	2×-10 = -20
x/2	0/2 = 0	2/2 = 1	-2/2 = -1	5/2 = 2.5	-10/2 = -5

3.3 Subscripts

In the last subsection we covered the concept of a variable. Since variables are represented by letters, we are limited to 52 symbols. This number considers both lowercase and uppercase. (Of course, it is actually less than this, since some letters have very similar symbols for uppercase and lowercase or resemble other reserved symbols.) This may seem like a lot of choices but as we go along we will start reserving different symbols for different concepts. For example, we will use s for speed, v for velocity, F for force, and many others.

One way to remedy this is to introduce another alphabet, such as Greek (ϕ, θ, ω and γ are some examples of Greek letters). We won't cover the concepts that use these letters, but you either have or will soon. (Ever heard of an alpha (α) particle or a beta (β) particle?)

This still does not help use to refer to multiple objects or different times. For example, consider a situation where we have two objects moving. How can we have different variables for each object's speed. The answer to this is the subscript.

One possible solution is to call one object "big" and the other one "small" (assuming that they have different mass or size). Then we could take the first letter from each object's name (b and s, respectively), and use those letters as a subscript. We would then have s_b and s_s as variables. They could be interpreted as follows; s_b is interpreted as the speed of the big object and s_s is interpreted as the speed of the small object.

One drawback of this approach is that different people will choose different styles of creating subscripts. For example, in the previous example, someone could choose to use b_s to mean the speed of the big object. With practice, it will become easier to interpret different styles of subscripts.

3.4 The Difference Operator

Since Physics is often concerned with change (as you will see), it is convenient to have an operator dedicated to change. This operator is called the difference operator. Its symbol is the Delta (Δ) symbol. Using subscripts, the difference operator can be written as shown in equation 8.

$$\Delta x = x_f - x_i \tag{8}$$

The way to read equation 8 is the change of x is equal to the final value of x minus the initial value of x. **Be very careful about the order.** It is a common mistake to get them backwards.

As an example, consider that your back account has thirty thousand dollars, you then get paid and your balance is now thirty-four thousand dollars. If we use B as the variable to represent your balance, then can find the difference as is shown in equation 9.

$$\Delta B = B_f - B_i = 34,000 - 30,000 = 4,000 \tag{9}$$

Therefore, you were paid four thousand dollars after taxes. Note that if you got the order backwards you would have gotten negative four thousand. That would mean that your balance dropped four thousand dollars. Hopefully you can see that that didn't happen, so that is a sign to check your work.

3.5 Order of operations

When evaluating more complicated expressions, such as in expression 7, we need rules to decide the order in which we evaluate the expression. Note that these rules are somewhat arbitrary. Mathematicians decided the order a long time ago, but they could have been different. Note that if they were different then the way we write expressions would also be different. The order is as follows.

1. Parentheses (perform any operations in parentheses first)
2. Exponents
3. Multiplication and division (recall that division is a type of multiplication, see section 2.2.2)
4. Addition and subtraction (recall that subtraction is a type of addition, see section 2.2.1)

As an example, let's evaluate expression 7 for the values from zero to two. Thee solution is shown in the table below where each column is a step by step solution. Note that you could do only one operation per line and still come to the correct answer.

t = 0	t = 1	t = 2	Description
$-16 \times 0^2 + 2 \times 0 + 7$	$-16 \times 1^2 + 2 \times 1 + 7$	$-16 \times 2^2 + 2 \times 2 + 7$	Substitute the value for t into the expression.
$-16 \times 0 + 2 \times 0 + 7$	$-16 \times 1 + 2 \times 1 + 7$	$-16 \times 4 + 2 \times 2 + 7$	Since no parentheses, evaluate the square.
$0 + 0 + 7$	$-16 + 2 + 7$	$-64 + 4 + 7$	Perform all multiplication or division.
7	-7	-53	Perform all addition or subtraction.

3.6 Equations

Simply put, an equation is two (or more expressions) set equal to each other. See the table below for examples.

Expression 1	Expression 2	Equation
x + 3	2x + 5	x + 3 = 2x + 5
4x + 5	2x + 3	4x + 5 = 2x + 3
2x + 10	5x - 2	2x + 10 = 5x - 2

There are two broad types of equations; linear and non-linear. We will mostly work with linear equations. A linear equation is an equation with no variables with exponents. Furthermore, linear equations can be divided into three classes; identity, contradiction and conditional. We will only deal with the conditional type. What the conditional type means is that there is one single value of the variable that makes the equality true.

The equality sign is an extremely strong statement in mathematics. It means that we can change both sides of the equation in many ways so long as we change both sides the same way.

For the purposes of this class, this means the four operations of arithmetic as well as squaring and square rooting. We will mostly be focusing on the four operations of arithmetic.

To be more concrete, consider the equation in the first row of the above table: $x + 3 = 2x + 5$. Since we can do anything to the equation so long as we do it to both sides, I could choose to subtract five. If I do so then the equation changes to $x - 2 = 2x$. Likewise, I could choose to subtract x from both sides. If I do then the equation becomes $-1 = x$. Rewritten, this becomes $x = -1$. Changing an equation like this is called solving the equation.

In general, solving an equation means reorganizing the equation such that the variable is set equal to something else. That is, the variable is on only one side of the equation. The way to do this requires practice. However, a guideline for solving linear equations is to first combine constants and variables. Then undo addition or subtraction by subtraction or addition, respectively. Then undo multiplication or division by division or multiplication, respectively. Consider the following two examples.

4x + 5 = 2x + 3	Since I first want to combine constants, I subtract 5 from both sides.
4x = 2x - 2	I now want to combine variables, so I subtract 2x from both sides.
2x = -2	I now want to undo multiplication, so I divide by 2 on both sides.
x = -1	This is my final answer.

However, we could choose to solve for an expression in an equation.

4 Units and Dimensions

4.1 Unit Conversions

4.1.1 Unit Conversion by Substitution

This method of unit conversion works by replacing a unit with an equivalent expression. For example, if we want to convert 3 yards to feet, we would use the conversion factor 1 yard = 3 feet and replace the unit "yards" with the expression "3 feet". Like so: 3 yards = 3 (3 feet) = 9 feet. Consider the following examples.

- Convert 15 hours to minutes. We know that there are 60 minutes in an hour, or stated algebraically, 1 hr = 60 min. Therefore;
  15 hr = 15 (60 min) = 15(60) min = 900 minutes

- Convert 8 pints to cups. We know that there are 2 cups in a pint, or written algebraically, 1 pint = 2 cups. Therefore;
  8 pints = 8 (2 cups) = 8(2) cups = 16 cups

- Convert 5 miles to yards. We know that there are 1760 yards in a mile, or stated algebraically, 1 mile = 1760 yards. Therefore;
  5 mi = 5 (1760 yards) = 5(1760) yards = 8800 yards

As of yet, we have only converted from large units to small units. We can convert from small units to large units using this method. However, we must solve our equation relating units for the other units. See below for some examples.

- Convert from 8 quarts to gallons. We know that there are 4 quarts in a gallon, or stated algebraically, 4 quarts = 1 gallon. However, we need to convert from quarts to gallons, not

gallons to quarts, so we solve the equation for quarts. Therefore, 1 quart = 1/4 gallons = 0.25 gallons. Therefore;
8 quarts = 8 (.25 gallons) = 8(.25) gallons = 2 gallons

- Convert 8100 minutes to hours. We know that there are 60 minutes in an hour, or stated algebraically, 60 minutes = 1 hour. However, we need to convert from minutes to hours, not hours to minutes, so we solve the equation for hours. 1 minute = 1/60 hours. Therefore;
8100 minutes = 8100 (1/60 hours) = (8100/60) hours = 135 hours

5 Motion

5.1 Velocity

Just like speed, there are three types of velocity; instantaneous velocity, average velocity and relative velocity

5.1.1 Instantaneous Velocity

Instantaneous velocity is the velocity that an object is moving at any particular moment in time.

Even if an object travels at a constant speed, its direction may be constantly changing. Consider a person running around a city block. The person will regularly change direction. In this case, the instantaneous velocity will change occasionally. For instance, the runner may start running west at 3 mph. Then he may turn and start running North at 3 mph. In each case, the speed is constant, but the direction is different.

As another example, consider a marble in a Roulette wheel. As the marble spins around the track, it has a speed that is near constant, but the direction is constantly changing. Therefore its instantaneous velocity is constantly changing.

Of course, the speed is free to change at the same time as the direction. Consider a pendulum, as the bob starts traveling downward from its highest position, it will speed up. However, it will also start to travel in a different direction. Therefore, both the speed and the direction are constantly changing in this scenario.

5.1.2 Average Velocity

Average velocity is the total displacement divided by the total time it took.

As an example, consider a NASCAR driver who has completed a loop around the track at the Atlanta Motor Speedway and is back where he started from. Suppose that his lap time is one minute. Since the distance around the track is 1.540 miles [3], his average speed is 1.540 miles/min or 92.4 mph. However, since he is back where he started, his displacement is zero. Therefore, his average velocity is zero.

5.1.3 Relative Velocity

There is no difference between relative speed and relative velocity except that relative velocity includes direction and speed doesn't include direction.

6 Laws of Motion

6.1 Free Body Diagrams

For an introduction to Free Body Diagrams, refer to the following link: https://www.bbc.co.uk/bitesize/guides/zpwhrwx/revision/3

6.2 Torsion force

A torsion force is a force which attempts to twist an object. Examples include automotive drive-shafts and constant velocity (CV) axles, door knobs, turbine shafts, ratchet extensions, motor shafts, pulley axles and more.

6.3 Friction

Refer to the pages from 165 to 170 and the section entitled "Friction always a Resistance" on page 171 in the book "An Elementary Text-book of Theoretical Mechanics".[2] This book is available at https://archive.org/details/anelementarytex00merrgoog/page/n173/mode/2up. Note that the coefficient of friction in the text is represented by ϕ whereas μ is more typically used today. Also feel free to ignore all references to or suggestions of trigonometry. We will not be using that material.

References

[1] Fisher, George and Schwatt, Isaac, *Higher Algebra*, Philadelphia, Pennsylvania, US: Norwood Press, 1901, https://archive.org/details/higheralgebra00schwgoog

[2] Merrill, George A, *An Elementary Text-book of Theoretical Mechanics*, New York, New York, US: American Book Company, 1905, https://archive.org/details/anelementarytex00merrgoog,

[3] https://www.nascar.com/tracks/. Accessed 2023-05-29.